Int. J. Heat Mass Transfer. Vol. 13., pp. 657–666. Pergamon Press 1970. Printed in Great Britain

ON BUBBLE GROWTH RATES

B. B. MIKIC, W. M. ROHSENOW and P. GRIFFITH
Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

(*Received* 21 *July* 1969 *and in revised form* 20 *October* 1969)

Abstract— A simple general relation for bubble growth rates in a uniformly superheated liquid was derived. The relation is valid in both regions: inertia controlled and heat diffusion controlled growth, respectively. The derived relation is compared with the existing experimental results for bubble growth in a uniformly superheated liquid with very good agreement.

The results are further extended to the bubble growth in a non-uniform temperature field which approximates the conditions present in a nucleate boiling from a heated surface.

NOMENCLATURE

A, a parameter, defined by equation (8);
a_l, thermal diffusivity of liquid;
B, a parameter, defined by equation (9);
b, a constant, defined by equation (7);
h_{fg}, latent heat of evaporation;
Ja, Jakob number;
k, thermal conductivity;
p, pressure;
R, bubble radius;
R^+, nondimensional bubble radius, defined by equation (12);
r, radial coordinate;
T, temperature;
ΔT, liquid superheat, $T_\infty - T_{sat}$ or $T_w - T_{sat}$;
t, time;
t^+, nondimensional time, defined by equation (12);
u, radial velocity.

Subscripts
b, bulk;
l, liquid;
v, vapor;
w, wall.

Greek letters
ρ, density of fluid;
σ, surface tension;
θ, subcooling factor, $(T_w - T_b)/(T_w - T_{sat})$.

INTRODUCTION

Bubble growth rates were extensively investigated in the last few decades. Generally, the work has been divided into the following two main regions: growth rates controlled by inertia forces, applicable in the range of a relatively low pressure and high Jakob numbers, e.g. Rayleigh [1], and growth rates for heat diffusion controlled growth, e.g. Plesset and Zwick [2], Forster and Zuber [3], Scrieven [4], Dergarabedian [5], Bankoff *et al.* [6]. The heat diffusion controlled growth was later extended to consideration of non-uniform temperature field effects, e.g. Griffith [7], Savic [8], Han and Griffith [9], Bankof and Mikesell [10], Zuber [11], Cole and Shulman [12], Van Stralen [13], Mikic and Rohsenow [14].

As the result of the above investigations, we have presently the two basic types of the growth rate relations each of which is applicable only to a given, though not distinctly defined, range. For a given set of bubble growth conditions, e.g. given fluid, pressure, liquid superheat, etc., we could not, *a priori*, tell in all cases which one of the two basic relations would apply unless we had some previous experimental results taken under similar conditions which identified the controlling mechanism for the bubble growth. The above uncertainty represents a serious disadvantage when one deals with liquids or conditions for which bubble growth data are not available (for example liquid metals).

Recently, Lien and Griffith [15] experimentally investigated bubble growth in uniformly superheated water covering the pressure range from 0·18 to 5·6 psia. They concluded that the bubble growth at very low pressures is controlled solely by inertia forces and that, as the pressure increases heat diffusion becomes a predominant factor, which at the upper part of their pressure range completely controlled the bubble growth. They also concluded that the interface resistance (kinetic effects) at the vapor–liquid interface is never a significant factor in bubble growth. This fact was never established before. The authors presented a growth curve in non-dimensional form showing that a single curve correlates all their experimental results.

Mainly motivated by the experimental results of the above investigation, this work develops one single analytical relation applicable in the entire range of bubble growth in a uniformly superheated liquid.

Problem statement

Consider a growing vapor bubble in a pool of liquid at a pressure p_∞ and initially uniform temperature T_∞. Let p_v and T_v represent the vapor pressure and the vapor temperature inside a growing bubble, respectively. Let us further assume that the vapor is in equilibrium with the liquid, hence p_v and T_v would represent a saturation state for the considered fluid.

In general, the vapor pressure and temperature, respectively, could lie in the following range: $p_\infty \leqslant p_v \leqslant p(T_\infty)$ and $T_{sat} \leqslant T_v \leqslant T_\infty$ where $p(T_\infty)$ is the vapor pressure corresponding to the saturation temperature T_∞ and T_{sat} is the saturation temperature corresponding to the pressure p_∞.

From the condition that $p_v \leqslant p(T_\infty)$ immediately follows that the growing bubble must be bigger than a critical size bubble for which the entire available pressure drop $[p(T_\infty) - p_\infty]$ would occur across the vapor–liquid interface due to the surface tension forces.

The conditions which determine the bubble growth can be stated as follows:

(i) The vapor pressure inside the growing bubble is related, in principal, to the motion of the vapor–liquid interface (and liquid) through the momentum equation.

(ii) The vapor temperature is related to the bubble growth rate through the energy balance requirement which relates the rate of bubble growth (vapor generation) to the heat flux at the vapor–liquid interface (and the temperature distribution in the liquid).

(iii) Finally, since the thermodynamic equilibrium is assumed, the vapor temperature is related to the vapor pressure through the Clausius–Clapeyron equation.

Principally, then, the above three conditions, which could be presented by three analytical relations, completely determine the bubble growth rate, i.e. R (bubble radius), p_v and T_v can be expressed as a function of time (for a given initial condition).

As it was pointed out earlier, presently the solutions for the bubble growth problem exists only for the two limiting cases, namely

(1) When the pressure inside the bubble was assumed to be the maximum possible pressure $p(T_\infty)$ and the growth was solved from the momentum equation, i.e. condition (i). The above approach would imply that $T_v = T_\infty$ i.e. that a negligible temperature potential between the liquid and the vapor $(T_\infty - T_v)$ is required to generate necessary amount of vapor for the particular bubble growth. This solution is known as inertially controlled growth.

(2) The second type of solutions assumes that the temperature inside the growing bubble is the minimum possible temperature, i.e. T_{sat}. This solution is derived only from condition (ii). The solution implies that the vapor pressure is p_∞ and hence, that a negligible amount of pressure difference $(p_v - p_\infty)$ is required to displace the liquid for the particular growth. This type of the solution is known as heat

diffusion controlled growth or the asymptotic solution.

The purpose of this work is to derive a solution which could be applicable to the entire range of possible cases.

General relations

In the following analysis it will be assumed that the bubble grows from $R = 0$, rather than from a critical radius R_c. This standard approach is already used on the two existing solutions with good results. Furthermore, the analysis will not use the equation of motion to relate vapor pressure to the liquid motion. Instead, the related equation of mechanical energy will be used. Neglecting irreversible conversion to internal energy, gravitational effects and work done by viscous forces, this approach consists of equating the total kinetic energy of the moving liquid at any time with the work done at the liquid boundaries. This approach is more convenient for the case of a spherical bubble growth attached to a surface, where the momentum equation is complicated due to the absence of spherical symmetry, but the value for the total kinetic energy exists in the literature.

Consider a growth of a spherical bubble which starts to grow at $t = 0$ at $r = 0$ (Fig. 1a). The equation of continuity, assuming that the liquid is incompressible and extends in all directions to infinity, is given as

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u) = 0 \tag{1}$$

where u is liquid velocity.

At the liquid–vapor interface ($r = R$), neglecting the mass transfer due to evaporation

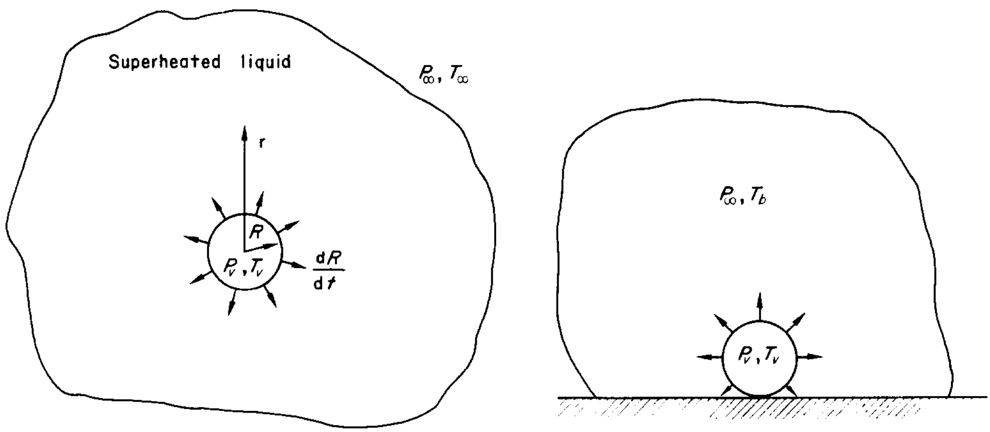
$$u_{r=R} = \frac{dR}{dt}.$$

Integrating equation (1) from $r = R$ to one obtains liquid velocity in terms of the interface velocity

$$u = \frac{dR}{dt}\left(\frac{R^2}{r}\right). \tag{2}$$

The total kinetic energy (K.E.) of the liquid mass at any time can be computed from relation (2) as follows:

$$\text{K.E.} = \frac{\rho_l}{2}\int_R^\infty u^2\, dV = \frac{\rho_l}{2}\int_R^\infty \left[\frac{dR}{dt}\left(\frac{R}{r}\right)^2\right]^2 4\pi r^2 dr$$



(a) (b)

FIG.

or after integration

$$\text{K.E.} = 2\pi\,\rho_l \left(\frac{dR}{dt}\right)^2 R^3. \qquad (3)$$

For a spherical bubble growth on a plane surface (Fig. 1b) the total kinetic energy has been calculated [16–17] as*

$$\text{K.E.} = 9{\cdot}35\,\rho_l \left(\frac{dR}{dt}\right)^2 R^3. \qquad (3a)$$

Assuming, for simplicity, that the gravity is not present—the assumption, as it can be easily shown, does not limit the results of the analysis only to the conditions of zero gravity—we can equate the total kinetic energy of the system (liquid mass) with the work done on its boundaries.

Taking that the bubble grows from $R = 0$–R, the net work to the liquid can be expressed as

$$W = 4\pi \int_0^R p_i R^2\,dR - \tfrac{4}{3}\pi\,p_\infty R^3. \qquad (4)$$

The first term on the right-hand side in the above equation represents the work done to the liquid on the liquid–vapor interface. p_i is the pressure on the liquid side of the interface, which can be related to the vapor pressure inside the growing bubble (p_v) as

$$p_v = p_i + \frac{2\sigma}{R}.$$

The second term on the right-hand side in relation (4) represents the work done by the liquid on the environment at p_∞ (neglecting the work done by the viscous forces on a heated surface for the case of the bubble growth on the surface). Substituting the above relation for p_i in equation (4), equating the latter with the total kinetic energy for the case of growth in an infinite pool of liquid, equation (3), and differentiating the obtained result with respect to R, one gets the known form of the equation of motion, i.e.

$$\rho_l\left[\frac{3}{2}\left(\frac{dR}{dt}\right)^2 + R\frac{d^2R}{dt^2}\right] = p_v - p_\infty - \frac{2\sigma}{R}. \qquad (5)$$

Again, as pointed out earlier, we will not use equation (5) in our analysis directly for the reasons already stated.

Neglecting the pressure drop across the liquid–vapour interface, the work done on the liquid follows from (4) as

$$W = 4\pi \int_0^R (p_v - p_\infty) R^2\,dR$$
$$= \frac{4}{3}\pi R^3\left[(p_v - p_\infty) - \frac{1}{4}\frac{dp_v}{dR}R + \frac{1}{4\times 5}\frac{d^2p_v}{dR^2}R^2 + \ldots\right]_R$$

where the last expression was obtained by successive integration by parts. Assuming that the variation in the vapor pressure for each individual bubble growth is not large, we can approximate the integral only with the first term on the right-hand side, i.e.

$$W \simeq \tfrac{4}{3}\pi R^3 (p_v - p_\infty) \qquad (6)$$

implying also, for variable p_v, that the total value of the integral for W comes predominantly from the contribution at the integration near the upper limit of the integral. Additional discussion of the error involved by the use of relation (6) is given later when overall results are considered.

Equating now the approximate expression for the net work done, equation (6), with the total kinetic energy of the liquid, equation (3), or (3a), one obtains:

$$\left(\frac{dR^2}{dt}\right) = b\,\frac{p_v - p_\infty}{\rho_l} \qquad (7)$$

where $b = 2/3$ for bubble growth in an infinite mass of liquid, and $b = \pi/7$ for a spherical bubble growing attached to a surface.

* 9·35 was taken from [16]. Slightly different coefficient (9·33) has been calculated by an approximate method in [17].

Bubble growth in a uniformly superheated liquid at T_∞

Using the Clausius–Clayeyron equation and linearizing it one can relate the pressure difference to the corresponding temperature difference as

$$\frac{p_v - p_\infty}{\rho_l} = \frac{T_v - T_{sat}}{\rho_l T_{sat}} \rho_v h_{fg}.$$

With the above relation (7) changes into

$$\left(\frac{dR}{dt}\right)^2 = A^2 \frac{T_v - T_{sat}}{\Delta T} \tag{8}$$

where

$$\Delta T = T_\infty - T_{sat} \quad \text{and} \quad A = \left(b \frac{h_{fg}\rho_v \Delta T}{\rho_l T_{sat}}\right)^{\frac{1}{2}}.$$

The bubble grows due to the evaporation of the liquid at the vapor–liquid interface. The heat required for the evaporation is supplied from the superheated liquid. The driving temperature potential between the liquid and vapor is $(T_\infty - T_v)$. For the cases when the vapor density is very small, relatively small evaporation will cause substantial bubble growth. So here very little temperature difference $(T_\infty - T_v)$ is needed. In the limit $T_v \to T_\infty$ and $T_v - T_{sat} = \Delta T$. With this one can integrate equation (8) directly, obtaining the well known Rayleigh solution for the bubble growth controlled by the inertia forces only, as

$$R = At.$$

In general, T_v is different from T_∞ and could take any value between the two limits of T_∞ and T_{sat}, depending on the particular conditions present in the considered growth.

The bubble growth in an initially superheated liquid due to a constant temperature difference $(T_\infty - T_v)$ was investigated, as pointed out earlier, by Plesset and Zwick [2], Scriven [4] and others. In [2] the following relation was developed

$$\frac{dR}{dt} = \frac{1}{2}\frac{B}{\sqrt{t}}\frac{T_\infty - T_v}{\Delta T}$$

$$= \frac{1}{2}\frac{B}{\sqrt{t}}\left(1 - \frac{T_v - T_{sat}}{\Delta T}\right) \tag{9}$$

where

$$B = \left(\frac{12}{\pi} a_l\right)^{\frac{1}{2}} Ja; \qquad Ja = \frac{\Delta T c_l \rho_l}{h_{fg}\rho_v};$$

a_l is the liquid thermal diffusivity.

If the variations in T_v for *a particular bubble* growth is not large during most of the time of its growth, then it would be reasonable to assume that relation (9) can be used for an approximate evaluation of vapor temperature required for a given growth. This relates T_v to (dR/dt). It can be shown that this approach is equivalent of assuming that the shape of the temperature profile inside the boundary layer around the growing bubble does not change in time (remains selfsymmetric) and that the thickness of this layer (penetration depth) is proportional to $(a_l t)^{\frac{1}{2}}$. Further discussion of the error involved in the above approximation is presented later in the general discussion of the results.

Thus, solving for $(T_v - T_{sat})/\Delta T$ from relation (9), and substituting the result in equation (8), one obtains the following

$$\frac{1}{A^2}\left(\frac{dR}{dt}\right)^2 + \frac{2\sqrt{t}}{B}\frac{dR}{dt} - 1 = 0. \tag{10}$$

Or after solving for dR/dt and expressing the result in a dimensionless form

$$\frac{dR^+}{dt^+} = (t^+ + 1)^{\frac{1}{2}} - (t^+)^{\frac{1}{2}} \tag{11}$$

where

$$\left.\begin{aligned} R^+ &= \frac{AR}{B^2} \\ \text{and} \quad t^+ &= \frac{A^2 t}{B^2}. \end{aligned}\right\} \tag{12}$$

Integrating (11) and setting $R^+ = 0$ at $t^+ = 0$, the general bubble growth relation was obtained as

$$R^+ = \tfrac{2}{3}[(t^+ + 1)^{\frac{3}{2}} - (t^+)^{\frac{3}{2}} - 1]. \quad (13)$$

For $t^+ \ll 1$ (13) simplifies into the Rayleigh solution

$$R^+ = t^+ \quad \text{or} \quad R = At. \quad (14)$$

For $t^+ \gg 1$ one gets from (13) the Plesset and Zwick relation

$$R^+ = \sqrt{(t^+)} \quad \text{or} \quad R = B\sqrt{t}. \quad (15)$$

The derived relation for the bubble growth, as it can be seen above, goes to the right limits on the both ends. This was expected since in the derivation the exact conditions on the both ends were built in.

Bubble growth in a non-uniform temperature field

For a bubble growth on a heated surface at a temperature T_w in a liquid at a temperature T_b and pressure p_∞, expression (8) is still valid but here we define ΔT as $\Delta T = T_w - T_{sat}$, where T_{sat}, as before, is saturation temperature corresponding to the pressure p_∞. The constant b in the expression for A, according to the previous discussion concerning the total kinetic energy for a spherical bubble growth on a surface, has a value of $\pi/7$.

Mikic and Rohsenow [14] considered bubble growth in a non-uniform temperature field. They used one-dimensional model corrected for sphericity in order to get in the limit the correct result for the bubble growth in a uniformly superheated liquid, equation (9). The model basically consists of the following two steps: (i) liquid at a uniform temperature T_b comes into contact with a surface at T_w and (ii) after time t_w the bubble forms and grows in the non-uniform temperature field obtained in the previous step during the time t_w. The vapor temperature inside the bubble is constant at T_v. The following result was obtained

$$p_v h_{fg} \frac{dR}{dt} = k_l \sqrt{3}\left[\frac{T_w - T_v}{(\pi a_l t)^{\frac{1}{2}}} - \frac{T_w - T_b}{[\pi a_l (t + t_w)]^{\frac{1}{2}}}\right] \quad (16)$$

or

$$\frac{dR}{dt} = \frac{1}{2}\frac{B}{\sqrt{t}}\left[\frac{T_w - T_v}{\Delta T} - \theta\left(\frac{t}{t + t_w}\right)^{\frac{1}{2}}\right]. \quad (16a)$$

t in the above relations is measured from the commencement of the bubble growth. B has the same value as in the previous section, with the understanding that ΔT in the Jakob Number and throughout this section is $T_w - T_{sat}$. θ represents the measure of the subcooling and it is defined as

$$\theta \equiv \frac{T_w - T_b}{T_w - T_{sat}}.$$

For the same reasons as used in the previous section, we assume that relation (16a) can be used to relate T_v to dR/dt for a particular bubble growth. Thus, calculating T_v from (16a), and substituting the result into (8), the following non-dimensional relation was obtained

$$\left(\frac{dR^+}{dt^+}\right)^2 + 2\sqrt{t^+}\frac{dR^+}{dt^+} - \left[1 - \theta\left(\frac{t^+}{t^+ + t_w^+}\right)^{\frac{1}{2}}\right] = 0$$

where R^+ and t^+ are defined by equation (12).

The above expression yields the bubble growth rates as

$$\frac{dR^+}{dt^+} = \left[t^+ + 1 + \theta\left(\frac{t^+}{t^+ + t_w^+}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} - (t^+)^{\frac{1}{2}}. \quad (17)$$

For $t_w^+ \to \infty$, the case of the growth in a uniformly superheated liquid, relation (17) reduces to expression (11).

Integration of equation (17) could be performed for given parameters θ and t_w^+. In the next section some results for $\theta = 1$ and different values of t_w^+ are presented. For cases where $t^+ \gg 1$ integration of (17) yields ([14]):

$$R^+ = (t^+)^{\frac{1}{2}}\left\{1 - \theta\left[\left(1 + \frac{t_w^+}{t^+}\right)^{\frac{1}{2}} - \left(\frac{t_w^+}{t^+}\right)^{\frac{1}{2}}\right]\right\}. \quad (18)$$

RESULTS AND DISCUSSION

Equation (13) is plotted in Fig. 2 (solid line denoted by $t_w^+ = \infty$ covering six orders of magnitude for R^+ range (from 10^{-4} to 10^2) and seven orders of magnitude for t^+ (from 10^{-4} to 10^3).

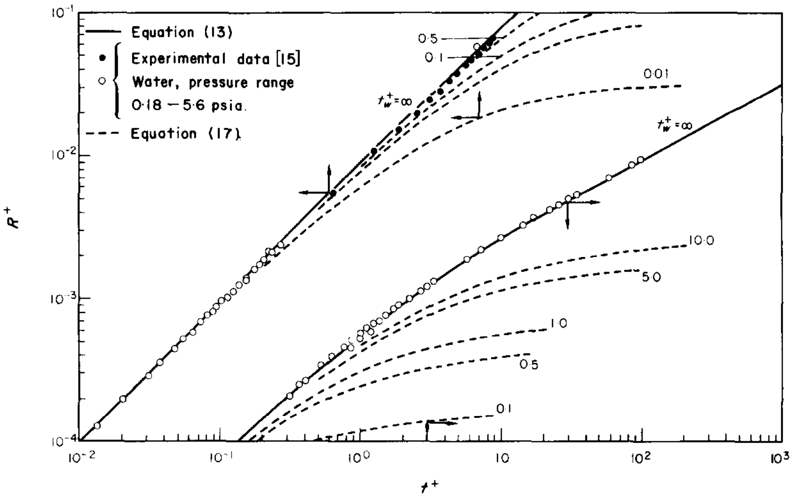


FIG. 2.

The experimental results of Lien and Griffith [15] for bubble growth rates in a uniformly superheated liquid (water) are presented on the figure for comparison. The experimental results comprise growth rates for six different bubbles, each at a different pressure.

As it can be seen from the figure, the agreement between the prediction, based on the present analysis, and the experimental results is very good. We can conclude, based on the above agreement, that relation (13) predicts bubble growth for each individual bubble in all of its recorded range of the growth. The analysis, however, will not give the correct prediction for initial stages of a bubble growth. For example, in Fig. 2 solid circles represent experimental points for a particular bubble growth. The first point corresponds to $t = 5{\cdot}5 \times 10^{-4}$ s, $R = 0{\cdot}587$ mm and the last point to $t = 7 \times 10^{-3}$ s, $R = 6{\cdot}88$ mm. The bubble did not approach its first recorded point along the prediction line from equation (13), since for this particular bubble the line left of its first recorded point would correspond to very small radii and actual times, where the assumptions incorporated in the analysis are not justifiable. In particular: (i) the analysis assumes that the bubble starts to grow from $R = 0$ at $t = 0$; however, as pointed out earlier, a bubble cannot exist in a thermodynamic equilibrium with the surrounding liquid for radii smaller than certain critical radius (R_c) determined by the fluid properties and the liquid superheat $[R_c = 2\sigma(T_{sat})/(h_{fg}\,\rho_v\,\Delta T)]$; (ii) in this work we also neglected the pressure drop across liquid–vapor interface (this assumption is also implicitly included in point (i) and this is not acceptable in the range of the bubble radii close to R_c; (iii) finally, our assumption involving the use of relation (9) (Plesset–Zwick equation) for the evaluation of the vapor temperature inside the growing bubble are not correct in the initial stages of the bubble growth.

Nevertheless, as it can be seen from the comparison, the uncertainty related to the early life history of a bubble did not noticeably affect the prediction for most of its life range.

The two critical assumptions used in this work are the one involved in derivation of equation (6) and the use of Plesset–Zwick relation to express T_v as a function of dR/dt. In the following, due to the significance of those assumptions, we pay an additional attention to them.

The first one introduced in the evaluation of the integral in expression (4) is equivalent, as can be seen by comparing relations (7) and (5), of assuming that the quantity

$$\frac{2}{3}\left|\frac{R(d^2R/dt^2)}{(dR/dt)^2}\right|$$

is negligible compared to unity. If the growth law in a particular region is $R \sim t^n$ then the above quantity reduces to

$$\frac{2}{3}\,\frac{1-n}{n}.$$

On the far left of our curve ($t^+ \ll 1$) it is seen that $n = 1$ and the above term is zero. In the region around $t^+ = 1$, where the both effects (inertia and heat diffusion) are equally important, growth rate is approximately $R \sim t^{\frac{3}{4}}$ and the above term has the value of 2/9; since this error is present only in the inertia contribution, the overall effect on the growth rate in this range should be less than indicated by the above result. In the region where $t^+ \gg 1$ the inertia effects are altogether negligible and therefore it is immaterial what is the relative significance of the omitted term.

To estimate the error involved in using the Plesset–Zwick relation for evaluation of the vapor temperature, one can employ integral technique, assuming the temperature profile which will give in the limit (when T_v is constant) the Plesset–Zwick relation, specifying the variable heat flux at the vapor–liquid interface as $\rho_v h_{fg}(dR/dt)$, and calculate the interface temperature from the obtained result. This was done and the result shows that the use of the Plesset–Zwick relation for the evaluation of T_v is equivalent of assuming that

$$\eta \equiv \left[\frac{1}{2t}\,\frac{R}{dR/dt}\right]^{\frac{1}{2}} \text{ is unity}$$

or

$$(T_\infty - T_v) = \eta(T_\infty - T_v)_{\text{Plesset–Zwick}}$$

Again, if the growth law in a certain range of parameters is given as $R \sim t^n$ one gets $\eta = (1/2n)^{\frac{1}{2}}$. On the right-hand side (large t^+) of our curve, when heat diffusion is the controlling factor, $n = \frac{1}{2}$ and, of course $\eta = 1$, i.e. no error is involved in using Plesset–Zwick relation. In the region where both mechanisms are equally important, $n = \frac{3}{4}$ and $\eta = 0{\cdot}82$. Hence, the error in evaluating $(T_\infty - T_v)$ is about 18 per cent. For $t^+ \ll 1$ the diffusion does not play any significant role in the bubble growth and the error involved in the evaluation of T_v is unimportant (although it is interesting to note that even there $\eta = 0{\cdot}71$).

In conclusion, then, we can state that all the assumptions involved in the derivation of the general growth rate relation, equation (13), did not affect substantially its accuracy in the entire range of interest, which is, what is most important, confirmed by the available experimental results.

In Fig. 2, results of integration of equation (17), for $\theta = 1$ and several values of t_w^+ (from $t_w^+ = 10$ to $t_w^+ = 0{\cdot}01$) are presented. It can be seen from the plot that the temperature nonuniformity could have strong effect on bubble growth. Experimental results for the growth in a non-uniform temperature field for which waiting time (t_w) was recorded exists only in the range of $t^+ \gg 1$. These results were already compared with the prediction, equation (18) in [14], with a good agreement.

Data for bubble growth on a heated surface at reduced pressure (this would be normally located in the range of t^+ around unity and less) do exist, e.g. Cole and Shulman [12], but without required information concerning waiting time. In general the results presented in [12] fall below the prediction from equation (11), approximately by a factor of two, suggesting that temperature non-uniformity and possibly subcooling had a strong effect on the growth in the reported experiments. This behavior is in qualitative agreement with the trends shown in Fig. 2, where, for example, one can see that waiting time of the same order of magnitude as departure time would reduce the bubble radius almost by a factor of two (compared with the case of $t_w^+ \to \infty$).

CONCLUSIONS

For a bubble growth in a uniformly superheated liquid the following relation, applicable in the entire range of the growth curve (including inertia controlled and diffusion controlled growth, respectively) was developed:

$$R^+ = \tfrac{2}{3}[(t^+ + 1)^{\frac{3}{2}} - (t^+)^{\frac{3}{2}} - 1]$$

where

$$R^+ = \frac{R}{B^2/A};\ t^+ = \frac{t}{B^2/A^2}$$

$$A = \left[b\frac{\Delta T h_{fg}\rho_v}{T_{sat}\rho_l}\right]^{\frac{1}{2}};\ B = \left[\frac{12}{\pi}Ja^2 a_l\right]^{\frac{1}{2}}$$

$b = \frac{2}{3}$ for bubble growth in an infinite medium; $b = \pi/7$ for bubble growth on a surface.

The analysis was also extended to consider effects of the non-uniform temperature field present in the case of a bubble growth on a heated surface.

REFERENCES

1. L. Rayleigh, Pressure due to collapse of bubbles, *Phil. Mag.* 94 (1917).
2. M. S. Plesset and S. A. Zwick, The growth of vapor bubbles in superheated liquids, *J. Appl. Phys.* **25**(4), 493 (1954).
3. H. K. Forster and N. Zuber, Growth of a vapor bubble in a superheated liquid, *J. Appl. Phys.* **25**(7), 474 (1954).
4. L. E. Scriven, On the dynamics of phase growth, *Chem. Engng Sci.* **10**, 1 (1959).
5. P. Dergarabedian, The rate of growth of vapor bubbles in superheated water, *J. Appl. Mech.* **20**, 537 (1953).
6. G. Birkhoff, R. S. Margulies and W. A. Horning, Spherical bubble growth, *Physics Fluids* **1**(3), 201 (1958).
7. P. Griffith, Bubble growth rates in boiling, *J. Heat Transfer* **80**, 721 (1958).
8. P. Savic, Discussion on bubble growth rates in boiling, *J. Heat Transfer* **80**, 5162 (1958).
9. C. Y. Han and P. Griffith, The mechanism of heat transfer in nucleate pool boiling, Parts I and II, *Int. J. Heat Mass Transfer* **8**, 887 (1965).
10. S. G. Bankoff and R. D. Mikesell, Growth of bubbles in a liquid of initially non-uniform temperature, *ASME Annual Meeting*, Paper No. 58-A-105, New York (1958).
11. N. Zuber, Vapor bubbles in non-uniform temperature field, *Int. J. Heat Mass Transfer* **2**, 83 (1961).
12. R. Cole and H. L. Shulman Bubble growth rates of high Jakob numbers, *Int. J. Heat Mass Transfer* **9**, 1377 (1966).
13. J. D. Van Stralen, The mechanism of nucleate boiling in pure liquids and in binary mixture, Parts I and II, *Int. J. Heat Mass Transfer* **9**, 995–1046 (1966).
14. B. Mikic and W. Rohsenow, Bubble growth rates in non-uniform temperature field, *Progress in Heat and Mass Transfer* **II**, 283 (1969).
15. Y. Lien and P. Griffith, The bubble growth at reduced pressure, to be submitted to *Int. J. Heat Mass Transfer*; also Y. Lien, Bubble growth rates at reduced pressure, Sc.D. Thesis, Mech. Eng. Dept., M.I.T. (Jan. 1969).
16. M. Shiffman and B. Friedman, On the best location of a mine near a sea bed, *Underwater Explosion Research*, Vol. II. U.S. Office of Naval Research (1950).
17. C. P. Witze, V. E. Schrock and P. L. Chambre, Flow about a growing sphere in contact with a plane surface, *Int. J. Heat Mass Transfer* **11**, 1637 (1968).

SUR LES VITESSES DE CROISSANCE DES BULLES

Résumé—Une relation générale et simple pour les vitesses de croissance des bulles a été obtenue dans un liquide uniformément surchauffé. La relation est valable dans deux régions: respectivement celle de la croissance contrôlée par l'inertie et celle de la croissance contrôlée par la diffusion de la chaleur. La relation obtenue est comparée avec un très bon accord avec les résultats expérimentaux existant pour la croissance des bulles dans un liquide surchauffé uniformément.

Les résultats sont étendus plus loin à la croissance des bulles dans un champ non uniforme de température qui rend compte d'une façon approchée des conditions existant dans une ébullition nucléée à partir d'une surface chauffée.

ÜBER BLASENWACHSTUMSRATEN

Zusammenfassung—Für die Blasenwachstumsraten in einer gleichmässig überhitzten Flüssigkeit wurde eine einfache allgemeine Beziehung abgeleitet. Die Beziehung ist für beide Gebiete, sowohl für das Trägheitskraft- wie auch für das Wärmediffusions- geregelte Wachstum gültig. Die abgeleitete Beziehung kann sehr gut mit den vorhandenen experimentellen Ergebnissen für das Blasenwachstum in einer gleichmässig überhitzten Flüssigkeit verglichen werden.

Die Ergebnisse erstrecken sich auch auf das Blasenwachstum in einem nicht einheitlichen Temperaturfeld, was angenähert den Bedingungen für das Blasensieden an einer beheizten Oberfläche entspricht.

О СКОРОСТИ РОСТА ПУЗЫРЬКОВ

Аннотация—Выведено простое общее соотношение для скорости роста пузырьков в однородно перегретой жидкости. Соотношение справедливо для двух областей: когда рост пузырьков определяется силами инерции и термодиффузией. Выведенное соотношение хорошо согласуется с имеющимися экспериментальными результатами по росту пузырьков в однородно перегретой жидкости.

Результаты распространены на случай роста пузырьков в неоднородном температурном поле, моделирующем условия, имеющие место при ядерном кипении на нагретой поверхности.